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Stochastically optimal active control of a smart truss structure under stationary random excitation

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Abstract

Optimization of the placement and feedback gains of an active bar in a closed-loop control system for random intelligent truss structures under stationary random excitation are studied in this paper. Based on maximization of dissipation energy due to the control action, a mathematical model with reliability constraints on the mean square value of the structural dynamic displacement and stress response is developed. The randomness of the physical parameters corresponding to the structural materials, geometric dimensions and structural damping are included in the analysis, and the applied forces are considered as stationary random excitation. The numerical characteristic of the stationary random responses of a stochastic smart structure is developed. Numerical examples of stochastic truss structures are presented to demonstrate the rationality and validity of the active control model, and some useful conclusions are obtained.

1. Introduction

Intelligent (smart) structures are becoming increasingly important for vibration and its control [1–3], as their characteristics can be self-modified during operation to improve their resistance to external disturbances. A piezoelectric (PZT) smart truss structure is used in spacecraft deployable

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antenna, large antennas, and other important large-scale truss structures, in which the PZT active bar can be used as both an actuator for vibration excitation, and sensor for vibration measurement. Optimal placement of the PZT active bar is an important factor in the process of the structural design phase, and its shape and vibration control. The location of active bars in the intelligent truss structure directly affects the validity of active vibration control.

In recent years, there has been much work published on intelligent structures. Chen et al. [4] investigated optimal placement of active and passive members in complex truss structures, and maximization of the cumulative energy dissipated over finite time intervals as the measure of optimality. Peng et al. [5] studied active position and vibration control of composite beams with distributed PZT sensors and actuators, with a finite element model based on third-order laminate theory. Suk et al. [6] introduced the Lyapunov control law for the slew maneuver of a flexible space structure by using a time-domain finite element analysis. To optimize the gain set of the control system, an energy-based performance index was adopted, and the gradients of the performance index with respect to each gain were derived. Ray [7] proposed a simple method for optimal vibration control of simply supported thin-laminated shells integrated with PZT layers.

To date, the majority of modelling on optimization of active vibration control using PZT smart structures have used deterministic models to model the dynamic response of smart structures, and optimal placement of the PZT actuators and sensors. In these cases, the structural parameters, applied loads and control forces are regarded as known parameters. However, deterministic models of the dynamic response associated with smart structures cannot reflect the influence of the randomness of the structural parameters. The dynamic response of an engineering structure can be sensitive to randomness in its parameters arising from variability in its geometric or material parameters, or randomness resulting from the assembly process and manufacturing tolerances. In addition, applied loads can be random process forces, such as wind, earthquakes and blast shock. The problem of stochastic smart structures subject to random applied excitation is of great significance in realistic engineering applications.

The dynamic response analysis of a closed-loop control system for an intelligent structure is an important segment in the process of its design and vibration control, in particular, to determine the optimal location of an active bar. It is only in recent years that the dynamic response of stochastic structures under random excitation has received research attention. Wall and Buchner [8] studied the dynamic effects of uncertainty in structural properties when the excitation is random by use of perturbation stochastic finite element method (PSFEM). Liu et al. [9] discussed the secular terms resulted from PSFEM in transient analysis of such a random dynamic system. Jensen and Iwan [10] examined the response of systems with uncertain parameters to random excitation by extending the orthogonal expansion method. Zhao and Chen [11] investigated the vibrational response of structures with stochastic parameters to random excitation using the dynamic Neumann stochastic finite element method, in which the random equation of motion for structure is transformed into a quasi-static equilibrium equation for the solution of displacement in time domain.

In this paper, optimization of the location of the active bar and feedback gain in PZT stochastic truss structures are investigated. The randomness of the physical parameters corresponding to the structural materials, geometric dimensions and structural damping are simultaneously considered. The applied force is taken as a stationary random excitation. Numerical expressions for the mean values of the natural frequencies, and displacement and stress responses of an intelligent truss structure are obtained. The performance function due to the control action is based on maximization of the dissipation energy. To formulate the optimal control problem, the algorithm for a linear quadratic regulator with output feedback has been employed in this paper. An optimal mathematical model with reliability constraints on the mean square value of structural dynamic displacement and stress response is developed. Numerical examples of stochastic truss structures are presented to demonstrate the rationality and validity of the active control model, and some useful conclusions are obtained.

2. Optimal mathematical model

2.1. Performance function

Following the finite element formulation described in Ref. [4], the equation of motion for an intelligent structure is given by

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{F_P(t)\} + [B]\{F_C(t)\}, \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are displacement, velocity and acceleration vectors, respectively. $\{F_P(t)\}$ is the load force vector generating the primary excitation. $\{F_C(t)\}$ is the control force vector. The matrix $[B]$ defines the location of the active bar on the smart structure under consideration. In the following analysis, the Wilson's damping hypothesis [12] is adopted. Using the modal expansion $\{u(t)\} = [\phi]\{z(t)\}$, the equation of motion takes the form

$$[I]\{\ddot{z}(t)\} + [D]\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^{\mathrm{T}}\{F_P(t)\} + [\phi]^{\mathrm{T}}[B]\{F_C(t)\}, \tag{2}$$

where $[D] = \mathrm{diag}\lfloor 2\zeta_j\omega_j \rfloor$, $[\Omega] = \mathrm{diag}[\omega_j^2]$ for $j = 1 \ldots n$. $[\phi] = \lfloor \phi_1 \cdots \phi_n \rfloor$ is the normal modal matrix of the structure, and ω_j, ζ_j are the jth-order natural frequency and damping ratio, respectively.

For active control of the truss bar, a velocity feedback control law is considered. Since each active bar can be considered as a collocated actuator/sensor pair, the output matrix is the transpose of the input matrix. The output vector $Y(t)$ and control force vector $\{F_C(t)\}$ can be, respectively, expressed as

$$Y(t) = [B]^{\mathrm{T}}[\phi]\{\dot{z}(t)\}, \tag{3}$$

$$\{F_C(t)\} = -[G]Y(t) = -[G][B]^{\mathrm{T}}[\phi]\{\dot{z}(t)\}, \tag{4}$$

where $[G] = \mathrm{diag}\{g_j\}$ is the gain matrix [4]. Substituting Eq. (4) into Eq. (2) yields the equation of the closed-loop system

$$[I]\{\ddot{z}(t)\} + ([D] + [\phi]^{\mathrm{T}}[B][G][B]^{\mathrm{T}}[\phi])\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^{\mathrm{T}}\{F_P(t)\}, \tag{5}$$

In the state–space representation, the equation of motion for the closed-loop system becomes

$$\{\dot{u}(t)\} = [A]\{u(t)\}, \tag{6}$$

where

$$\{u\} = \{z(t)\ \ \dot{z}(t)\}^{\mathrm{T}}, \tag{7}$$

$$[A] = \begin{bmatrix} 0 & [I] \\ -[\Omega] & -\left([D] + [\phi]^{\mathrm{T}}[B][G][B]^{\mathrm{T}}\right)[\phi] \end{bmatrix}. \tag{8}$$

Both the optimal location of the active bar, and the optimal gain of the closed-loop control system are determined such that the total energy dissipated in the system is maximized. The total energy dissipated in the system is taken as the performance and it can be expressed as [13]

$$J = \int_0^\infty \{\dot{z}(t)\}^{\mathrm{T}}[\phi]^{\mathrm{T}}([D] + [B][G][B]^{\mathrm{T}})[\phi]\{\dot{z}(t)\}\,\mathrm{d}t. \tag{9}$$

Eq. (9) can also be expressed as [14]

$$J = \{u(0)\}^{\mathrm{T}} \int_0^\infty e^{[A]^{\mathrm{T}}t}[Q]e^{[A]t}\,\mathrm{d}t\{u(0)\}, \tag{10}$$

where $[Q] = \begin{bmatrix} [\Omega] & 0 \\ 0 & [I] \end{bmatrix}$. Making use of the method described in the Ref. [13], the performance function can be expressed as

$$J = \mathrm{tr}[W], \tag{11}$$

where the matrix $[W]$ can be obtained by solving the Lyapunov equation [13]

$$[A]^{\mathrm{T}}[W] + [W][A] = [Q]. \tag{12}$$

2.2. Optimal mathematical model

For the smart truss structure with random parameters, and where the load is a stationary random excitation, an optimization program is written with reliability constraints that implements the following steps. For a fixed gain ($g = g_j$), the optimal location of the active bar (that is, the optimal $[B]$ matrix) is obtained such that the total energy dissipated J is maximized. After the optimal placement of the active bar is determined, the feedback gain is then optimized. This is achieved by calculating the mean square displacement for each kth dof and mean square dynamic stress for each eth element. Reliability constraints are placed on the mean square displacement and stress, respectively, as follows:

$$R^*_{\psi^2_{\sigma e}} - P_r\left\{\psi^{2*}_{\sigma e} - \psi^2_{\sigma e} \geqslant \delta\right\} \leqslant 0, \quad e = 1, 2, \ldots, n, \tag{13}$$

$$R^*_{\psi^2_{uk}} - P_r\left\{\psi^{2*}_{uk} - \psi^2_{uk} \geqslant \delta\right\} \leqslant 0, \quad k = 1, 2, \ldots, n, \tag{14}$$

$$[B] \subset [B^*], \quad [G] < [G^*], \tag{15}$$

$[B]$ and $[G]$ are the design variables. $R^*_{\psi^2_{\sigma e}}$ and $R^*_{\psi^2_{uk}}$ are given values of reliability of the mean square stress and displacement responses, respectively. $P_r\{\cdot\}$ is the reliability obtained from the actual calculation. $\psi_{\sigma e}^{2*}$ and ψ_{uk}^{2*} are given limit values of the mean square stress and displacement responses, respectively. In this model, $[B]$, $[G]$, $R^*_{\psi^2_{\sigma e}}$, $R^*_{\psi^2_{uk}}$, $P_r\{\cdot\}$, $\psi_{\sigma e}^{2*}$ and ψ_{uk}^{2*} can be random variables or deterministic values. $\psi_{\sigma e}^{2}$ and ψ_{uk}^{2} are the mean square dynamic stress of the eth element, and displacement of the kth dof, respectively. δ is the given allowable deviation in order to avoid failure of the structure, which is produced by the lack of strength or stiffness. $[B^*]$ and $[G^*]$ are the upper bounds of $[B]$ and $[G]$, respectively.

In above model, the dynamic stress and response constraints are expressed by the probability form, which make the optimal problem difficult to solve. For this reason, the reliability constraints are transformed into normal constraints by means of the second-order moment theory on the reliability [15]. Hence, the reliability constraints equation (13) and (14) can be, respectively, expressed as

$$\beta^*_{\psi^2_{\sigma e}} - \frac{\mu_{\psi^{2*}_{\sigma e}} - \mu_{\psi^2_{\sigma e}} - \delta_{\psi^2_{\sigma e}}}{(\sigma^2_{\psi^{2*}_{\sigma e}} + \sigma^2_{\psi^2_{\sigma e}})^{1/2}} \leqslant 0, \quad e = 1, 2, \ldots, n, \tag{16}$$

$$\beta^*_{\psi^2_{uk}} - \frac{\mu_{\psi^2_{uk*}} - \mu_{\psi^2_{uk}} - \delta_{\psi^2_{uk}}}{(\sigma^2_{\psi^2_{uk*}} + \sigma^2_{\psi^2_{uk}})^{1/2}} \leqslant 0, \quad k = 1, 2, \ldots, n, \tag{17}$$

where $\beta^*_{\psi^2_{\sigma e}} = \Phi^{-1}(R^*_{\psi^2_{\sigma e}})$ and $\beta^*_{\psi^2_{uk}} = \Phi^{-1}(R^*_{\psi^2_{uk}})$ are the given reliability of the mean square value of the dynamic stress response and displacement of the kth dof, respectively. $\Phi^{-1}(\cdot)$ denotes the inverse function of the normal distribution of random variables. $\mu_{\psi^{2*}_{\sigma e}}$ and $\sigma^2_{\psi^{2*}_{\sigma e}}$ are the limit values for the mean value and variance of the mean square stress of the eth element $\psi^{2*}_{\sigma e}$, respectively. $\mu_{\psi^{2*}_{uk}}$ and $\sigma^2_{\psi^{2*}_{uk}}$ are the limit values for the mean value and variance of the mean square displacement of the kth dof ψ^{2*}_{uk}, respectively. $\mu_{\psi^2_{\sigma e}}$ and $\sigma^2_{\psi^2_{\sigma e}}$ are the mean value and variance of the mean square dynamic stress, respectively. $\mu_{\psi^2_{uk}}$ and $\sigma^2_{\psi^2_{uk}}$ are the mean value and variance of the mean square displacement of the kth dof, respectively. $\delta_{\psi^2_{\sigma e}}$ and $\delta_{\psi^2_{uk}}$ are the given allowable deviations of the mean square value of structural stress and displacement response, respectively. The mean square displacement for the kth dof and mean square stress for the eth element of the smart truss structure under consideration subject to a random stationary load force will be derived in the proceeding sections.

3. Stationary random response of the closed-loop control system

Suppose that there are n elements in the smart truss structure under consideration. In the structure, any element can be taken as either a passive or active bar, where a PZT bar is used as the active bar. The stiffness matrix $[K]$ and the mass matrix $[M]$ of the smart truss structure in global coordinates can be expressed as

$$[K] = \sum_{e=1}^{n}[K_e] = \sum_{e=1}^{n}\left\{\left[\theta\frac{E^P_e A^P_e}{l^P_e} + (1-\theta)\frac{c_{33e} + (e_{33e})^2/\varepsilon_{33e}}{l^C_e}A^C_e\right][T]\right\}, \tag{18}$$

$$[M] = \sum_{e=1}^{n}[M_e] = \sum_{e=1}^{n}\left\{\frac{1}{2}(\theta\rho_e^P A_e^P l_e^P + (1-\theta)\rho_e^C A_e^C l_e^C)[I]\right\}, \tag{19}$$

where θ is a Boolean algebra value defined by the following: when $\theta = 0$, the mixed element is a PZT active element bar; when $\theta = 1$, the mixed element is a passive element bar. $[K_e]$ is the stiffness matrix of the eth element, $[M_e]$ is the mass matrix of the eth element. ρ_e^P, A_e^P and l_e^P are the density, cross-sectional area and length, respectively, of the eth passive bar element. ρ_e^C, A_e^C and l_e^C are the density, cross-sectional area and length, respectively, of the eth active bar element. E_e^P is the Young's modulus of the eth passive bar element. c_{33e}, e_{33e} and ε_{33e} are the Young's modulus, PZT force/electrical constant and dielectric constant respectively of the eth active bar element [16]. $[I]$ is a sixth-order identity matrix, and $[T]$ is a 6×6 transformation matrix that relates the parameters of the eth truss element to those of a global coordinate system. E_e^C is the generalized elastic modulus of the PZT active bar which considers the mechanic–electronic coupling effect, and is given by

$$E_e^C = c_{33e} + (e_{33e})^2/\varepsilon_{33e}. \tag{20}$$

Substituting Eq. (20) into Eq. (18) yields

$$[K] = \sum_{e=1}^{n}[K_e] = \sum_{e=1}^{n}\left\{\left[\theta\frac{E_e^P A_e^P}{l_e^P} + (1-\theta)\frac{E_e^C A_e^C}{l_e^C}\right][T]\right\}. \tag{21}$$

In the closed-loop control system, since the control force $\{F_c(t)\}$ is determined by the applied force $\{F_c(t)\}$, the control force is a random force vector, and these two variables have full positive correlation. Let

$$\{P(t)\} = \{F_P(t)\} + [B]\{F_C(t)\} \tag{22}$$

Eq. (1) can be re-written as:

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\}. \tag{23}$$

Eq. (23) is a set of coupled differential equations. Its formal solution can be obtained in terms of the decoupling transform and Duhamel integral [17], that is

$$\{u(t)\} = \int_0^t [\phi][h(\tau)][\phi]^{\mathrm{T}}\{P(t-\tau)\}\,\mathrm{d}\tau, \tag{24}$$

$[h(t)]$ is the impulse response function matrix of the structure, and can be expressed as

$$[h(t)] = \mathrm{diag}\{h_j(t)\}, \tag{25}$$

$$h_j(t) = \begin{cases} \frac{1}{\omega_j'}\exp(-\zeta_j\omega_j t)\sin\omega_j' t, & t \geqslant 0, \\ 0, & t < 0, \end{cases} \quad j = 1, 2, \ldots, s, \tag{26}$$

where $\omega_j' = \omega_j(1-\zeta_j^2)^{1/2}$. From Eq. (24), the correlation function matrix of the displacement response of the structure can be obtained

$$[R_u(\varepsilon)] = E\left(\{u(t)\}\{u(t+\varepsilon)\}^{\mathrm{T}}\right) = \int_0^t \int_0^t [\phi][h(\tau)][\phi]^{\mathrm{T}}[R_P(\tau-\tau_1+\varepsilon)][\phi][h(\tau_1)]^{\mathrm{T}}[\phi]^{\mathrm{T}}\,\mathrm{d}\tau\,\mathrm{d}\tau_1, \quad (27)$$

where $[R_u(\varepsilon)]$ is the correlation function matrix of the displacement response of the structure, $[R_P(\tau-\tau_1+\varepsilon)]$ is the correlation function matrix of $\{P(t)\}$. By performing a Fourier transformation to $[R_u(\varepsilon)]$, the power spectral density matrix of the displacement response $[S_u(\omega)]$ can be obtained as follows:

$$[S_u(\omega)] = [\phi][H(\omega)][\phi]^{\mathrm{T}}[S_P(\omega)][\phi]\left[H^*(\omega)\right][\phi]^{\mathrm{T}}, \quad (28)$$

where $[S_p(\omega)]$ is the power spectral density matrix of $\{P(t)\}$. $\lfloor H^*(\omega)\rfloor$ is the conjugate matrix of $[H(\omega)]$, where $[H(\omega)]$ is the frequency response function matrix of the structure, and can be expressed as

$$[H(\omega)] = \mathrm{diag}\left[H_j(\omega)\right], \quad (29)$$

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + \mathrm{i}2\zeta_j\omega_j\omega}, \quad j = 1, 2, \ldots, s, \quad (30)$$

where $\mathrm{i} = \sqrt{-1}$ is the complex number. Integrating $[S_u(\omega)]$ within the frequency domain, the mean square value matrix of the structural displacement response $\left[\psi_u^2\right]$ can be obtained as

$$\left[\psi_u^2\right] = \int_0^\infty [S_u(\omega)]\,\mathrm{d}\omega = \int_0^\infty [\phi][H(\omega)][\phi]^{\mathrm{T}}[S_P(\omega)][\phi]\left[H^*(\omega)\right][\phi]^{\mathrm{T}}\,\mathrm{d}\omega. \quad (31)$$

The mean square displacement of the kth dof becomes

$$\psi_{uk}^2 = \vec{\phi}_k \int_0^\infty [H(\omega)][\phi]^{\mathrm{T}}[S_P(\omega)][\phi]\left[H^*(\omega)\right]\,\mathrm{d}\omega\vec{\phi}_k^{\mathrm{T}}, \quad k = 1, 2, \ldots, n, \quad (32)$$

where $\vec{\phi}_k$ is the kth line vector of the modal matrix $[\phi]$. Using the relationship between node displacement and element stress, the stress response of the eth element in the truss structure can be expressed as

$$\left\{\sigma_e(t)\right\} = E_e[B_1]\left\{u_e(t)\right\}, \quad e = 1, 2, \ldots, n, \quad (33)$$

where $\{u_e(t)\}$ is the displacement of the nodal point of the eth element, $[B_1]$ is the element's strain matrix. From Eq.](33), the correlation function matrix of the eth element stress response $[R_{\sigma e}(\tau)]$ can be obtained by

$$[R_{\sigma e}(\tau)] = E\left(\left\{\sigma_e(t)\right\}\left\{\sigma_e(t+\tau)\right\}^{\mathrm{T}}\right) = E_e[B_1][R_{ue}(\tau)][B_1]^{\mathrm{T}}E_e. \quad (34)$$

Furthermore, the power spectral density matrix of the stress response of the eth element $[S_{\sigma e}(\omega)]$ can be obtained

$$[S_{\sigma e}(\omega)] = E_e[B_1][S_{ue}(\omega)][B_1]^{\mathrm{T}}E_e. \quad (35)$$

Finally, the mean square value matrix of the eth element stress response $\left[\psi_{\sigma e}^2\right]$ becomes

$$\left[\psi_{\sigma e}^2\right] = E_e[B_1]\left[\psi_{ue}^2\right][B_1]^{\mathrm{T}}E_e. \quad (36)$$

4. Numerical characteristics of the stationary random response

4.1. Numerical characteristics of the natural frequency random variable

The following parameters corresponding to ζ_j, ρ_e^P, A_e^P, l_e^P, E_e^P, ρ_e^C, A_e^C, l_e^C and c_{33e} are simultaneously considered as random variables. From Eq. (20), it can be easily observed that E_e^C is a random variable. The randomness of physical parameters and geometrical dimensions will result in randomness of the matrices $[K]$ and $[M]$, and consequently the natural frequencies ω_j. Computational expressions for the mean value $\mu_{\omega j}$ and mean variance $\sigma_{\omega j}$ of the jth natural frequency ω_j in terms of its random variables have been obtained by means of the algebra synthesis method, and are reported in Ref. [18].

4.2. Numerical characteristics of the stationary random response of the closed-loop system of the stochastic intelligent structure

The randomness of the structural damping, natural frequencies and excitation will result in randomness in the structural dynamic responses of the closed-loop control system, corresponding to the displacement and dynamic stress. In this section, expressions for the numerical characteristics corresponding to the mean value and mean variance of the structural stationary response random variables are derived.

From Eq. (32), by means of the random variable's functional moment method [19], the mean value $\mu_{\psi_{uk}^2}$ and mean variance $\sigma_{\psi_{uk}^2}$ of the mean square displacement for the kth dof value can be obtained as

$$\mu_{\psi_{uk}^2} = \mu_{\vec{\phi}_k} \int_0^{\omega_c} \mu_{[H(\omega)]}\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\,\mathrm{d}\omega\,\mu_{\vec{\phi}_k^{\mathrm{T}}}, \quad k = 1,2,\ldots,n, \tag{37}$$

$$\begin{aligned}\sigma_{\psi_{uk}^2} = \mu_{\vec{\phi}_k}\Bigg\{\int_0^{\omega_c}\Big(&\mu^2_{[H(\omega)]}\Big(\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\Big)^2\sigma^2_{[H^*(\omega)]} + \sigma^2_{[H(\omega)]}\Big(\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\Big)^2\mu^2_{[H^*(\omega)]}\\ &+\sigma_{[H(\omega)]}\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\sigma_{[H^*(\omega)]}\Big)\mathrm{d}\omega\Bigg\}^{1/2}\mu_{\vec{\phi}_k^{\mathrm{T}}}, \quad k = 1,2,\ldots,n,\end{aligned} \tag{38}$$

where

$$\sigma_{[H(\omega)]} = \mathrm{diag}\left\{\frac{\left\{\left[\left(2\mu_{\omega_j} + \mathrm{i}2\mu_{\zeta_j}\omega\right)\sigma_{\omega_j}\right]^2 + \left[\left(\mathrm{i}2\mu_{\omega_j}\omega\right)\sigma_{\zeta_j}\right]^2\right\}^{1/2}}{\left(\mu^2_{\omega_j} - \omega^2 + \mathrm{i}2\mu_{\zeta_j}\mu_{\omega_j}\omega\right)^2}\right\}, \quad j = 1,2,\ldots,s, \tag{39}$$

where μ_{ζ_j} and σ_{ζ_j} are the mean value and mean variance of ζ_j. From Eqs. (37) and (38), the variation coefficient $v_{\psi_{uk}^2}$ of the random variable ψ_{uk}^2 can be obtained as the ratio of the mean variance to the mean value of the mean square displacement

$$v_{\psi^2_{uk}} = \frac{\sigma_{\psi^2_{uk}}}{\mu_{\psi^2_{uk}}}. \tag{40}$$

From Eq. (36), and by means of the algebra synthesis method [18], expressions for the numerical characteristics of the mean square stress for the eth element are obtained as

$$\mu_{[\psi^2_{\sigma e}]} = (\mu^2_E + \sigma^2_E)[B_1]\mu_{[\psi^2_{ue}]}[B_1]^{\mathrm{T}}, \quad e = 1, 2, \ldots, n, \tag{41}$$

$$\sigma_{[\psi^2_{\sigma e}]} = \left\{ (\mu^2_E + \sigma^2_E)^2 \left([B_1]\sigma_{[\psi^2_{ue}]}[B_1]^{\mathrm{T}}\right)^2 + (4\mu^2_E\sigma^2_E + 2\sigma^4_E)\left([B_1]\mu_{[\psi^2_{ue}]}[B_1]^{\mathrm{T}}\right)^2 + (4\mu^2_E\sigma^2_E + 2\sigma^4_E)([B_1]\sigma_{[\psi^2_{ue}]}[B_1]^{\mathrm{T}})^2 \right\}^{1/2}, \quad e = 1, 2, \ldots, n. \tag{42}$$

where $\mu_{[\psi^2_{\sigma e}]}$ and $\sigma_{[\psi^2_{\sigma e}]}$ are the mean value and mean variance of the mean square stress for the eth element, respectively. From Eqs. (41) and (42), the variation coefficient of the mean square value of the eth element stress response $v_{[\psi^2_{\sigma e}]}$ can be found

$$v_{[\psi^2_{\sigma e}]} = \frac{\sigma_{[\psi^2_{\sigma e}]}}{\mu_{[\psi^2_{\sigma e}]}}, \qquad e = 1, 2, \ldots, n. \tag{43}$$

5. Computational results

To illustrate the method, a 35 bar planar smart truss structure shown in Fig. 1 is used. A ground-level acceleration acts on the structure [19]. The material properties of the active and passive bars are given in Table 1.

In order to solve the optimal problem, two steps are adopted [20]. In the first step, the reliability constraints of dynamic stress and displacement are neglected, and the feedback gains are kept constant. Then, each element bar is taken as an active bar in turn and the corresponding performance function value is calculated. Based on the computational results for the dissipated energy, the optimal location of the active bar can be determined. In the second step, after the optimal placement of the active bar is obtained, the reliability constraints are imposed, and the optimization of feedback gain, that is, minimization of feedback gain will be developed.

5.1. Optimal placement of the active bar

For the first step, and letting the closed-loop control system feedback gains be $g = g_j = 50$, each element bar is taken as active bar in turn; the corresponding performance function value is given in Table 2.

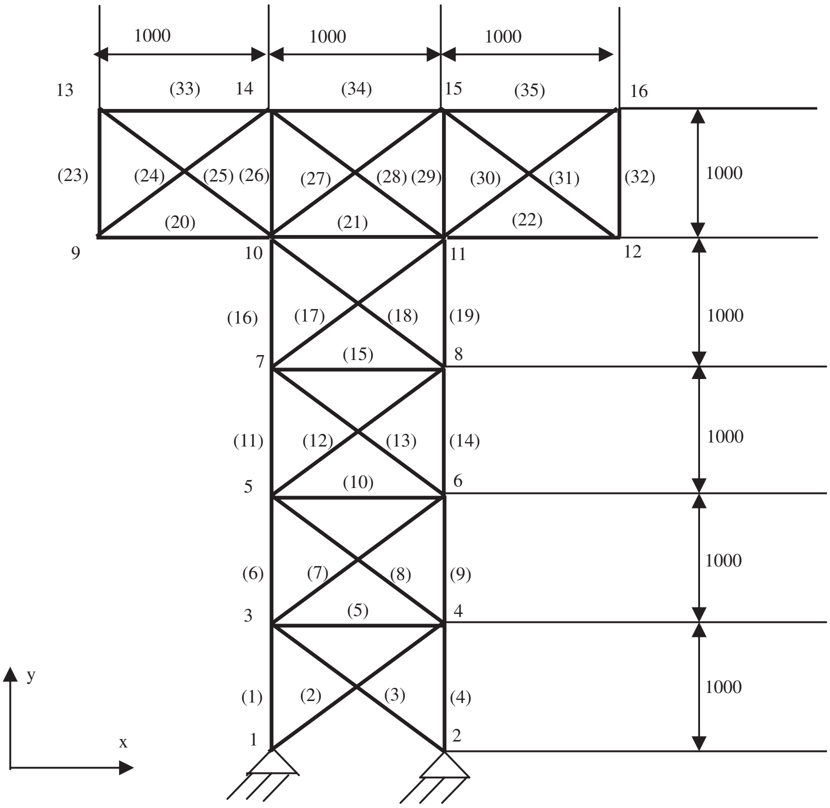

Fig. 1. 35 bar planar smart truss structure (units: mm)

Table 1
Physical parameters of the smart truss structure

	Active bar (PZT-4)	Passive bar (steel)
Mean value of mass density ρ (kg/m^3)	7600	7800
Mean value of elastic modulus c_{33} (N/m^2)	8.807×10^{10}	2.1×10^{11}
Piezoelectric force/electric constant e_{33} (C/m^2)	18.62	—
Dielectric constant ε_{33} (C/V m)	5.92×10^{-9}	—
Cross-sectional area A (m^2)	3.0×10^{-4}	3.0×10^{-4}

From Table 2, it can be seen that if the first or fourth element is used as the active bar, the active control performance of the smart truss structure is the best. The effect of active vibration control of the smart truss structure is the worst if the 23rd or 32nd element is used as the active bar. These results are not surprising since the control performance is the greatest when the active bar is closest to the primary ground excitation of the truss structure.

Table 2
Computational results of the performance function ($g = 50$)

Element	1	2	3	4	5	6	7	8	9
Value of J	178.31	169.55	169.55	178.31	157.27	145.64	139.26	139.26	145.64
Element	10	11	12	13	14	15	16	17	18
Value of J	131.18	120.44	108.97	108.97	120.44	98.71	90.28	81.33	81.33
Element	19	20	21	22	23	24	25	26	27
Value of J	90.28	63.22	65.85	63.22	17.12	43.67	39.75	58.19	49.58
Element	28	29	30	31	32	33	34	35	
Value of J	49.58	58.19	39.75	43.67	17.12	28.93	31.08	28.93	

5.2. Optimization of the feedback gain

In order to assess the control performance with the reliability constraints imposed and optimization of the feedback gain, the control results using the 1st and 32nd elements as the active bar respectively are compared. The structural parameters (material properties, geometric dimensions, structural damping) and the limit values of the mean square stress and displacement, $\psi_{\sigma e}^{2*}$ and ψ_{uk}^{2*}, are all taken to be random variables, where $\mu_{\psi_{\sigma e}^{2*}} = 2000\,\mathrm{MPa}^2, \mu_{\psi_{uk}^{2*}} = 3\,\mathrm{mm}^2$ and $R^*_{\psi_{\sigma e}^2} = R^*_{\psi_{uk}^2} = 0.95$. Values from both deterministic and random models were obtained. In the deterministic model, the mean values of the random variables are unity, and their mean variance is zero. The optimal results for the feedback gains, and the mean displacement and stress responses are given in Table 3, where $R_{\psi_{\sigma e}^2} = P_r\left\{\psi_{\sigma e}^{2*} - \psi_{\sigma e}^2 \geqslant \delta\right\}$ and $R_{\psi_{uk}^2} = P_r\left\{\psi_{uk}^{2*} - \psi_{uk}^2 \geqslant \delta\right\}$. Results for two random models are presented, in which the variation coefficients of all random variables is equal to 0.02 in the first random model, and 0.2 in the second random model. In addition, in order to verify our method, stationary random responses obtained using the Monte-Carlo simulation (MCS) method are also presented in Table 3.

From Table 3, it can be seen easily that the optimal results of the feedback gains obtained by the method proposed in this paper is in excellent agreement with that of the random structural stationary random responses analyzed by the MCS method, by which the validity of our method is verified. The optimal results of the deterministic and random models are different, and the optimal value of feedback gain increases when the randomness of the structural parameters increases. The results show that the areas of the truss structure where the most energy is stored are the optimal location of an active bar in order to maximize its damping effect.

6. Conclusions

Energy dissipation in a large smart truss structure has been maximized in order to determine the optimal location of a single PZT active element. Results show that the effectiveness of using the active element is strongly dependent on its location in the truss structure with respect to the primary load excitation. The effect of randomness of the structural parameters corresponding to

Table 3
Computational results of the feedback gains

Design variables	1st element used as the active bar: Original value	Determinate model	First random model	Second random model	32^{nd} element used as the active bar: Original value	Determinate model	First random model	Second random model
G	50	58.08	83.19	102.26	50	87.31	113.15	142.57
G^a			83.22[a]	102.29[a]			98.41[a]	127.50[a]
$\mu_{\psi^2_{\sigma e}}$ (MPa^2)	2103.9	1999.6	1582.3	1253.8	2239.7	1999.3	1582.8	1252.2
$\mu_{\psi^2_{\sigma e}}$ [a] (MPa2)			1582.9	1254.4[a]			1583.7[a]	1253.3[a]
$\mu_{\psi^2_{uk}}$ [a] (mm2)	2.9435	2.8442	2.3735	1.9026	3.1333	2.8451	2.3739	1.9007
$\mu_{\psi^2_{uk}}$ [a] (mm2)			2.3746[a]	1.9035[a]			2.3748[a]	1.9017[a]
$R_{\psi^2_{\sigma e}}$ [a]		0.47	0.98	0.98		0.47	0.98	0.98
$R_{\psi^2_{uk}}$ [a]		0.50	0.95	0.95		0.50	0.95	0.95

[a]Dynamic analysis by the MCS method.

the material properties, geometric dimensions and structural damping on the feedback gain was also examined.

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